Implications of $\Gamma(Z \to b\bar{b})$ for Supersymmetry Searches and Model-Building

James D. Wells[1], Chris Kolda[2], and G. L. Kane[3]

Randall Physics Laboratory, University of Michigan,
Ann Arbor, MI 48109–1120, USA

Abstract

Assuming that the actual values of M_t at FNAL and of $\Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ at LEP are within their current 1σ reported ranges, we present a No-Lose Theorem for superpartner searches at LEP II and an upgraded Tevatron. We impose only two theoretical assumptions: the Lagrangian is that of the Minimal Supersymmetric Standard Model (MSSM) with arbitrary soft-breaking terms, and all couplings remain perturbative up to scales $\sim 10^{16}$ GeV; there are no assumptions about the soft supersymmetry breaking parameters, proton decay, cosmology, etc. In particular, if the LEP and FNAL values hold up and supersymmetry is responsible for the discrepancy with the Standard Model prediction of $\Gamma(Z \to b\bar{b})$, then we must have charginos and/or top squarks observable at the upgraded machines (for LEP the superpartner threshold is below $\sqrt{s} = 140$ GeV). Furthermore, little deviation from the Standard Model is predicted within "super-unified" supersymmetry, so these models predict that the discrepancy between experiment and the Standard Model prediction for $\Gamma(Z \to b\bar{b})$ will fade with time. Finally, it appears to be extremely difficult to find any unified MSSM model, regardless of the form of soft supersymmetry breaking, that can explain $\Gamma(Z \to b\bar{b})$ for large $\tan\beta$; in particular, no model with $t - b - \tau$ Yukawa coupling unification appears to be consistent with the experiments.

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[1]E-mail: `jwells@umich.edu`

[2]E-mail: `kolda@umich.edu`

[3]E-mail: `gkane@umich.edu`

1 Introduction

Recent results from Fermilab [1] indicate that the top quark is rather heavy ($M_t = 174 \pm 17$ GeV), while recent results from LEP [2] indicate that $R_b \equiv \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ might be inconsistent (up to 2.5σ) with such a heavy top. In this letter we assume that both measurements are correct, and point out the rather powerful implications this has if nature is supersymmetric.

First, we give a brief discussion of the Standard Model (SM) prediction for R_b which is approximately 2 to 2.5σ away from the latest experimental measurement. We then consider the effect of supersymmetry (SUSY) on this process. In particular, we consider both the MSSM and the popular "super-unified" approach to supersymmetric model building. We demonstrate a No-Lose Theorem for discovery of superpartners at coming collider upgrades given 1σ experimental bounds on R_b and M_t, and a very minimal set of theoretical assumptions.

In considering the question of the R_b discrepancy, one can take either of two attitudes. Perhaps the R_b measurement is finally the one which directly demonstrates the existence of physics beyond the Standard Model; if so, its implications for the discovery of supersymmetry are dramatic (assuming SUSY is the origin of the deviation). Yet, as we will discuss in this paper, the "super-unified" SUSY models produce values for R_b near those of the SM, not large enough to explain the R_b discrepancy. Thus even if SUSY is the correct theory, it is not unlikely that the measurements of R_b will approach the SM expectation as systematic effects are more fully understood.

2 The Standard Model Prediction for $\Gamma(\mathbf{Z} \to \mathbf{b\bar{b}})$

The ratio R_b is very sensitive to vertex corrections involving a heavy top quark. Within the Standard Model these corrections are negative and grow like m_t^2. (This can be seen best in the 't Hooft-Feynman gauge where the $\phi^+ \bar{t} b$ coupling is proportional to m_t.) On the other hand, the m_t-dependent oblique corrections are to a good approximation universal and therefore largely cancel out in the R_b ratio. This effectively isolates the vertex corrections, thereby providing both an excellent test of the SM's self-consistency and a place to search for new physics beyond the reach of current experiments.

Using the program ZØPOLE [3] we have calculated R_b in the on-shell scheme for $4.5 \leq M_b \leq 5.3$ GeV as a function of the top quark pole mass, M_t. In Fig. 1 we have plotted the experimental values (and their 1σ ranges) for M_t and R_b. We also show the ZØPOLE calculation of the SM prediction as the shaded region in the figure. The SM prediction as quoted by LEP [2], which is calculated by ZFITTER in the on-shell scheme, falls on the right edge of the shaded region. To be conservative we utilize the rightmost edge of the SM prediction for R_b in all our calculations and comparisons since it lies closest to the the experimental measurement.

With an SM prediction in hand we now can compare with LEP's measurement of R_b. We use $R_b = 0.2208 \pm 0.0024$ [2]. The uncertainty includes the quoted statistical and systematic errors added in quadrature. One sees from Fig. 1 that $R_b^{\text{theory}} = 0.2158$ for $M_t = 174$ GeV. This is approximately 2σ away from the above quoted experimental measurement.
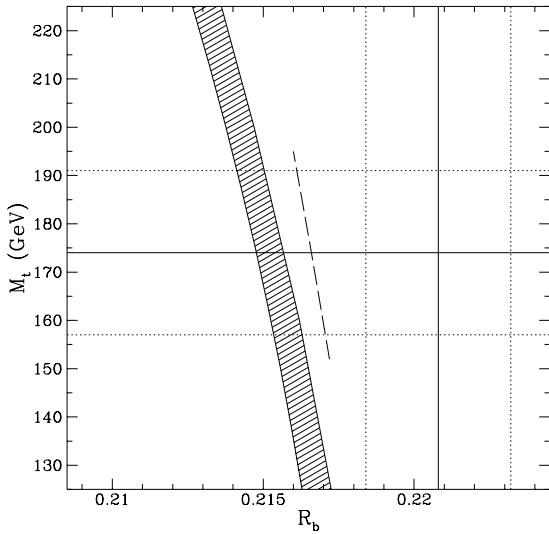
Figure 1: The SM prediction for R_b. The experimental central values of R_b and M_t (along with their 1σ bounds) are designated by straight solid (and dotted) lines. The shaded region represents the SM prediction for R_b from ZØPOLE, allowing for the b-quark mass to vary within the range $4.5 \leq M_b \leq 5.3\,\text{GeV}$. The curved dashed line represents the "line of closest approach" for the CMSSM to the experimental R_b value.

Disagreement of experiment and theory for other values of M_t can likewise be read off from Fig. 1. Since a heavy top forces R_b downward, a rather large positive contribution to R_b from new physics clearly is required to lift R_b to the reported central value, given CDF's measurement of a heavy top quark mass.

We understand of course that the measured R_b may decrease as systematic effects are better understood. However, if R_b is the long-awaited deviation from the SM that heralds the onset of new physics, its implications for SUSY are dramatic. More precisely, the question we consider here is the following: What implications are there for supersymmetry if the true value of R_b *is* within one standard deviation of the current measurement at LEP and the true value of M_t *is* within one standard deviation of the current measurement at FNAL? Henceforth we premise the true value of R_b and M_t to be within $R_b = 0.2208 \pm 0.0024$ and $M_t = 174 \pm 17\,\text{GeV}$ respectively, and investigate the consequences of this statement. We find several surprising results.

3   Supersymmetry mass bounds from $\Gamma(\text{Z} \to \text{b}\bar{\text{b}})$

In a supersymmetric theory there are additional corrections to the $Zb\bar{b}$ vertex which scale like m_t^2 from the charged Higgs–top quark loops and the chargino–top squark loops. Furthermore, if $\tan\beta$ is very large then, for example, the pseudoscalar Higgs (A^0) coupling to $b\bar{b}$ is proportional to $m_b \tan\beta$ and the bottom squark–b–neutralino $(\tilde{b}\bar{b}\chi^0)$ coupling similarly is proportional to $m_b/\cos\beta$. For $\tan\beta \gtrsim 40$ these contributions from "neutral exchanges"

can be sizable. The supersymmetric electroweak vertex corrections to the $Z b \bar{b}$ vertex at the Z pole have been analyzed by several authors [4, 5, 6, 7]. We primarily draw from the formulas of Ref. [6] (see Appendix) in order to calculate the supersymmetric contributions to R_b.

We will now investigate the Minimal Supersymmetric Standard Model (MSSM), which we define as the minimal (but most general) $SU(3) \times SU(2) \times U(1)$ gauge invariant Lagrangian containing the particle content of the SM, but which is supersymmetric (up to soft-breaking terms) and R-parity conserving.

Despite the large number of unknown soft-breaking parameters within the MSSM, any individual process usually depends only on a small subset. Thus it is with R_b, where only 12 unknown parameters of the MSSM enter. We will show that the experimental values for R_b may be used to constrain certain of these parameters to ranges that can be easily probed in the near future. Since the SM prediction for R_b is below the experimental one, the value which becomes most important to us for the present analysis is $R_b^{\max}$, the maximum value which R_b can take given any set of inputs and constraints.

The independent parameters which enter the calculation of R_b are $\tan \beta$, M_1, M_2, and μ (from which one gets the chargino and neutralino masses and mixings), the physical top and bottom squark masses and their mixing angles ($m_{\tilde{t}_{1,2}}$, $m_{\tilde{b}_{1,2}}$, $\theta_{\tilde{b}}$, and $\theta_{\tilde{t}}$), the pseudoscalar Higgs mass, and $\widetilde{V}_{tb}$ (the super-CKM angle between the bottom and top squarks). In theory, determining $R_b^{\max}$ for any given set of assumptions is very difficult, for it will be a complicated function of all the free parameters which enter the process. Luckily, we can separate the dependences on many of the parameters and work with them independently. For example, $R_b(\widetilde{V}_{tb})$ is always maximal when $|\widetilde{V}_{tb}| = 1$, regardless of the values of the other parameters.

Some parameters are *almost* separable. The top squark mixing parameter, $\theta_{\tilde{t}}$, tends to maximize $R_b(\theta_{\tilde{t}})$ for values which are small and negative. Yet the choice $\theta_{\tilde{t}} = 0$ always produces a near-maximal R_b, up to corrections of order 0.01σ, far too small for us to worry about here. Having now chosen $\theta_{\tilde{t}} = 0$ one then finds that $\tilde{t}_2 (= \tilde{t}_L)$ decouples and its mass is no longer one of the parameters on which R_b depends for low to intermediate $\tan \beta$. Also in this region of $\tan \beta$, the bottom squark–neutralino contributions decouple completely, leaving R_b independent of $m_{\tilde{b}_{1,2}}$, $\theta_{\tilde{b}}$, and M_1.

Finally, R_b is most strongly dependent on the masses of the light chargino(s), light top squark(s), and for large $\tan \beta$, neutral Higgs bosons. Lighter top squarks simply give larger contributions to R_b. The chargino contributions are much more complicated since they induce a local maximum of R_b right at $m_{\chi_1^{\pm}} = \sqrt{s}/2 = m_Z/2$ where R_b can be very large. Unfortunately, current bounds on the chargino mass fall right at $m_Z/2$; were these mass bounds a few GeV higher, much stricter bounds, for example on $m_{\tilde{t}_1}$ and $\tan \beta$, could be determined. This will be demonstrated explicitly later. Finally, the dependence on m_{A^0} changes sign as $\tan \beta$ increases, weakly favoring large m_{A^0} (actually, large $m_{H^{\pm}}$) for small to moderate $\tan \beta$, but strongly favoring small m_{A^0} for large $\tan \beta$. One has in total twelve parameters, though only seven of them are relevant at low to intermediate values of $\tan \beta$; with any fewer we lose complete generality.

3.1 The No-Lose Theorem

One may summarize the nature of our No-Lose Theorem thusly: SUSY is a decoupling theory. That is, if SUSY is to make measurable contributions to the radiative corrections of SM processes, the mass scale of the SUSY partners must be near to the scale of the SM physics being studied. Only for masses of spartners near to the current experimental limits can one expect to notice their effects through loops in SM diagrams. As these masses increase, the SM prediction for any given quantity is again realized. Thus, if we are to explain the discrepancy between the experimental and theoretical values for R_b using SUSY, the SUSY mass scale cannot be large.

That said, it remains to be seen what kind of numerical bounds can be placed on the masses of the sparticles entering into the SUSY contributions to R_b. The final result of such an analysis yields this: *If the discrepancy in R_b is to be explained by the MSSM, then direct observation of superpartners at LEP II or an upgraded Tevatron will occur.* Strict bounds may be placed on chargino and top squark masses if SUSY is to explain the value for R_b: $m_{\chi_1^\pm} < 85\,\mathrm{GeV}$ and $m_{\tilde{t}_1}$ or $m_{\tilde{b}_1} < 165\,\mathrm{GeV}$. Stronger bounds can also be found. For example, for $\tan\beta \lesssim 30$, $\min(m_{\chi_1^\pm}, m_{\tilde{t}_1}) < 65\,\mathrm{GeV}$. Under additional constraints, bounds exist also on m_{A^0}, $m_{\chi_1^0}$, and $\tan\beta$. This theorem holds under the following set of assumptions which we will examine below: *(i)* the true value for R_b is within 1σ of quoted LEP measurements, *(ii)* the true value for M_t is within 1σ of quoted CDF measurements, *(iii)* contributions from the MSSM are responsible for the difference between the actual and theoretical SM values, *(iv)* the Yukawa couplings of the MSSM remain perturbative up to scales $\sim 10^{16}\,\mathrm{GeV}$ (a so-called perturbatively valid theory), and *(v)* various experimental lower bounds on sparticle masses from direct searches.

Let us examine these conditions. The very first condition is also the one least trusted by us. Current LEP bounds on R_b are 2 to 2.5σ from theoretical expectations; however, if the LEP measurement of R_b migrates over time to the SM value, our theorem will of course cease to be meaningful.

The second condition is important mostly for the 1σ lower bound on M_t. In the SM as M_t decreases, the prediction for R_b increases such that the discrepancy between experiment and theory lessens. Also, when coupled with the requirement of perturbative validity (condition *(iv)* above), the lower bound on M_t places a lower bound on $\tan\beta$. This lower bound on $\tan\beta$ is necessary for the existence of upper mass bounds (see below).

The third condition listed above is self-explanatory.

The fourth condition of perturbative validity is necessary in order to place upper and lower bounds on $\tan\beta$. The requirement that the top, bottom, and tau Yukawa couplings remain perturbative up to a scale $\sim 10^{16}\,\mathrm{GeV}$ places limits on $\tan\beta$ of

$$\sin\beta > \frac{M_t}{M_{t0}} \quad \text{and} \quad \tan\beta \lesssim 60$$

where typically $190 \lesssim M_{t0} \lesssim 200\,\mathrm{GeV}$. These limits are necessary in order to gain mass bounds on the chargino and top squark. This can be seen in Fig. 2, where the maximum value for R_b is plotted against $\tan\beta$ for two choices of $m_{\chi_1^\pm}$. Clearly as $\tan\beta \to 0$ and $\tan\beta \to \infty$, R_b can become large even for heavy charginos and top squarks.
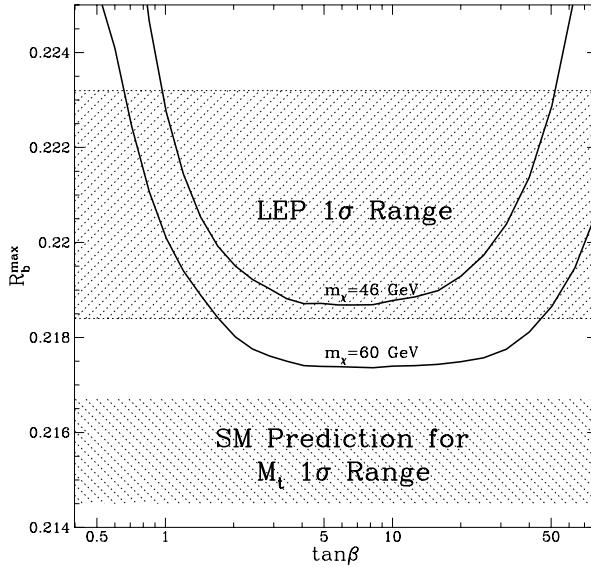
Figure 2: The dependence of $R_b^{\max}$ on $\tan\beta$. The maximum possible value for R_b obtainable as a function of $\tan\beta$ is plotted for $m_{\chi_1^\pm} = 46\,\text{GeV}$ (upper line) and $m_{\chi_1^\pm} = 60\,\text{GeV}$ (lower line). The upper hatched region is the experimental 1σ range for R_b, while the lower range represents the SM range consistent with the 1σ bounds for M_t.

The fifth and final "condition" is the set of experimental lower bounds that we apply to the masses of the MSSM. We take $m_{\chi_1^\pm} > 46\,\text{GeV}$ and the rather conservative $m_{\widetilde{t}_1} > 36\,\text{GeV}$ [8]. We take lower bounds on the pseudoscalar mass from the non-observation of $Z \to h^0 A^0$ (low $\tan\beta$) and $Z^0 \to A^0 b\bar{b}$ (high $\tan\beta$) at LEP, with A^0 then decaying into $\tau^+\tau^-$. These bounds are $\tan\beta$-dependent. For $1 \lesssim \tan\beta \lesssim 3$¡, $m_{A^0} > 20\,\text{GeV}$ [9]; for $\tan\beta \gtrsim 30$, $m_{A^0} > 60\,\text{GeV}$ [10].

In the following sections we demonstrate how the bounds on chargino and top squarks are determined through the set of five conditions above.

One should note that the bounds given above can be easily tightened. In particular, if M_t is found to be larger than its 1σ lower bound then our bounds will become stronger. For $M_t = 174$, the separate upper bounds on $m_{\chi_1^\pm}$ and $m_{\widetilde{t}_1}$ decrease to $63\,\text{GeV}$ and $77\,\text{GeV}$, with a corresponding decrease in the minimum of the two. Bounds on $\tan\beta$ will also exist if values of $m_{\chi_1^\pm}$ near to $46\,\text{GeV}$ are ruled out experimentally, as shown by Fig. 2; there the lower line represents $R_b^{\max}$ for $m_{\chi_1^\pm} > 60\,\text{GeV}$, in which case we see that the entire region of $2 \lesssim \tan\beta \lesssim 40$ is ruled out by the 1σ R_b bounds.

3.2   Low and Intermediate $\tan\beta$ Region

Consider first the case of low $\tan\beta$ (*i.e.*, $\tan\beta \lesssim 5$). Here one finds that R_b increases monotonically as $\tan\beta$ decreases (see Fig. 2). Therefore $R_b^{\max}(\tan\beta)$ is found at the boundary where $\sin\beta = M_t/M_{t0}$. For intermediate $\tan\beta$ (*i.e.*, $5 \lesssim \tan\beta \lesssim 30$), $R_b^{\max}$ increases with

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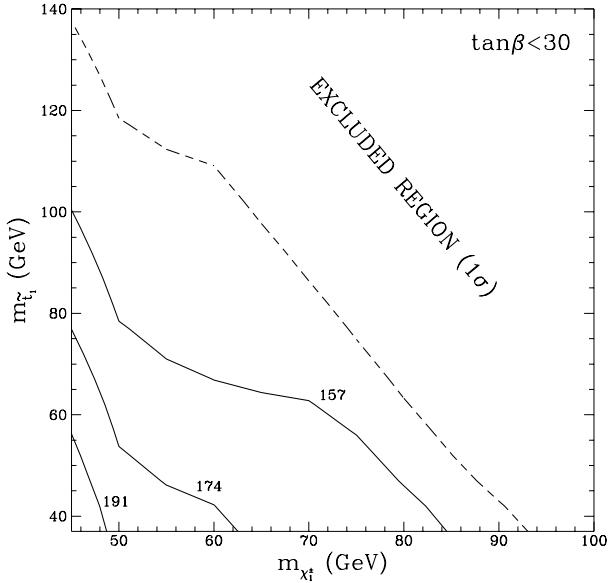
Figure 3: Upper bounds on $(m_{\chi_1^\pm}, m_{\tilde{t}_1})$ such that R_b will fall within 1σ of experiment, for $\tan\beta < 30$. Each solid line represents a different value for M_t (157, 174, and 191 GeV) such that only the region below and to the left of the lines are consistent with the R_b measurement, *i.e.*, $R_b \geq 0.2184$. The region above and to the right of the $M_t = 157$ GeV line is excluded to 1σ. In order to show how the limits are altered if the experimental value for R_b changes, we plot a dashed line representing the upper bounds for $M_t = 174$ GeV and $R_b \geq 0.2172$.

$\tan\beta$ but remains below the values obtained at very small $\tan\beta$. Therefore, one may simplify the analysis in this region by only considering the lowest possible value of $\tan\beta$ for a given M_t.

In Fig. 3 we have plotted contours of $R_b^{\max} = 0.2184$ (the 1σ lower experimental bound) against $m_{\chi_1^\pm}$ and $m_{\tilde{t}_1}$, for $m_t = 157$, 174, and 191 GeV. Recall that $R_b^{\max}$ is the largest value that the theory can produce, regardless of the values of the other parameters not shown in the plot. To be conservative, we take $M_{t0} = 205$ GeV, thereby allowing smaller $\tan\beta$ (and thus larger $R_b^{\max}$) for a given top quark mass. We emphasize that what is shown are the maximum values of $m_{\tilde{t}_1}$ and $m_{\chi_1^\pm}$ that can give R_b within 1σ of its reported value. In the figure, the regions to the left and below each line are compatible with the 1σ bounds on R_b; the regions above and to the right are excluded.

One can read the central result of the No-Lose Theorem from this graph: by combining the 1σ bounds on the top quark mass from CDF and on R_b from LEP, one can place upper bounds of 85 GeV on the mass of the lighter chargino and 100 GeV on the mass of the lighter top squark in the low to intermediate $\tan\beta$ region. However, one or the other must always be lighter than about 65 GeV. For M_t above 157 GeV, these limits become stronger. These bounds are to our knowledge the strongest experimental bounds on the MSSM to date. They do not depend on any scheme for soft-breaking parameters, on constraints from dark matter or proton decay, or any assumptions other than those few listed. They imply that with an upgrade in energy to $\sqrt{s} = 130$ GeV at LEP a chargino or a top squark must

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be found, and with an upgrade in luminosity at the Tevatron both the chargino and the top squark should be found. Therefore, the "1σ" ranges of chargino and top squark masses as shown in Fig. 3 could be completely probed if $\tan\beta \lesssim 30$.

Further, if $m_{\chi_1^\pm} \gtrsim 60\,\mathrm{GeV}$ and $\tan\beta \lesssim 40$, then $\tan\beta$ is restricted to be close to 1. Therefore, the mass of the lightest Higgs approaches zero at tree level, most of its mass then being due to radiative corrections. Since we bound the lighter top squark mass, the leading term in the corrections to the Higgs mass (which goes as m_t^4) is determined up to the value of the heavier top squark. Ignoring contributions due to top squark mixing, one finds that for $m_{\widetilde{t}_2} < 1\,\mathrm{TeV}$, $m_{h^0} < 75\,\mathrm{GeV}$; likewise for $m_{\widetilde{t}_2} < 2\,\mathrm{TeV}$, $m_{h^0} < 80\,\mathrm{GeV}$. Thus in the low to intermediate $\tan\beta$ regime, h^0 may also be accessible at LEP or FNAL.

3.3 The High $\tan\beta$ Region

Large values of R_b can also be attained for high $\tan\beta$. Once again, it is at the perturbative edge ($\tan\beta \simeq 60$) that we obtain the largest R_b^{max}. Interestingly, it is also this region of very high $\tan\beta$ that is motivated by $t - b - \tau$ Yukawa unification within minimal SO(10) models. In this region interactions proportional to $m_b \tan\beta$ become comparable to and possibly more significant than the m_t dependent interactions. Therefore, neutralino–bottom squark loops and neutral Higgs–bottom quark loops must be considered.

Four of the independent parameters of our model which could be ignored in the low $\tan\beta$ region, namely the bottom squark masses and mixing angle, and M_1, must now be included when we maximize R_b. This significantly complicates the process and the demonstration of our theorem, so we again separate those variables that can be taken independent from the others. One simplification would be to set $M_1 \simeq \frac{1}{2}M_2$, as indicated by wide classes of supergravity and superstring scenarios. We have taken this simplification as well-motivated and base most of our numerical results in the high $\tan\beta$ limit on it; however, we have checked that perturbing the ratio of M_1 to M_2 by an additional factor of two in either direction only changes our calculations of R_b^{max} by less than 0.1σ.

The bottom squark masses and mixing provide another complication. Our calculations indicate that R_b is maximized when both bottom squarks are light, near to their lower bound which we take to be $45\,\mathrm{GeV}$. The squarks are then nearly degenerate in mass and the mixing angle becomes arbitrary.

The Higgs sector behaves in the high $\tan\beta$ limit very differently than in the opposite limit. Unlike in the low $\tan\beta$ case, a light pseudoscalar Higgs yields an overall large *positive* contribution to R_b. The contribution of the light charged Higgs is still negative, but is overwhelmed by a large contribution from neutral Higgs–bottom quark loops which increases with $\tan\beta$. Therefore, we must reintroduce the pseudoscalar mass as a variable when working in this region. However the experimental constraint that $m_{A^0} > 60\,\mathrm{GeV}$ for $\tan\beta > 30$ [10] leads to chargino bounds at large $\tan\beta$ which are tighter than those at small $\tan\beta$.

How do discovery limits compare in the high $\tan\beta$ region? Most significantly, one finds that $m_{\chi_1^\pm} \lesssim 70\,\mathrm{GeV}$, a tighter bound than existed in the low $\tan\beta$ limit. However, it is not solely the chargino–top squark loops that are responsible for this bound, but also the neutralino–bottom squark loops. As the chargino mass becomes larger and its contributions decouple, R_b^{max} plummets towards the SM value, thereby placing a bound on $m_{\chi_1^\pm}$. Yet as

$\tan \beta$ increases, the neutralino contributions can be sizable, bringing R_b back into agreement with experiment. Thus there is really a bound on a combination of the neutralino and chargino masses. But the appearance of the same μ in both sectors means that bounds can be placed on each individually, leading to the $70 \, \text{GeV}$ for the chargino given above. The comparable neutralino bound is then approximately $67 \, \text{GeV}$. If the 1σ lower bound of R_b shifted to 0.2172, then the bounds on the chargino and neutralino would relax to 98 and $95 \, \text{GeV}$ respectively.

The same interplay we found between chargino and neutralino masses is of course also found in the top and bottom squark sectors. Under the assumption of nearly degenerate bottom squarks, one finds that either a bottom *or* a top squark must be lighter than $165 \, \text{GeV}$ ($85 \, \text{GeV}$) for $\tan \beta \leq 60$ and $m_{\chi_1^\pm} \geq 45 \, \text{GeV}$ ($m_{\chi_1^\pm} \geq 60 \, \text{GeV}$), a weaker bound than was found in the low $\tan \beta$ limit. But unlike the case for the charginos and neutralinos, no individual bounds exist on the top and bottom squarks alone.

Finally, there are also the contributions due to A^0–bottom quark loops. Once again an upper bound can be placed on m_{A^0} consistent with R_b if $m_{\chi_1^\pm} \geq 60 \, \text{GeV}$. Here we find that $m_{A^0} < 95 \, \text{GeV}$ is necessary. Thus, light pseudoscalars are implied by R_b for high $\tan \beta$, once one constrains charginos to be slightly heavier than the current bound of $46 \, \text{GeV}$.

4 Implications for the Super-Unified MSSM

Much work has been completed recently by a number of groups on the phenomenology of "super-unified" minimal SUSY (see Ref. [11] and references therein). These models are constructed under the assumption of not only gauge coupling unification at some high scale, but also unification of various soft mass parameters in the MSSM Lagrangian (a common gaugino and a common scalar mass). One then connects these high scale assumptions to low-energy phenomenology through the renormalization group equations (RGE's) of the parameters, under the condition that electroweak symmetry-breaking occurs radiatively at scales $\sim m_Z$.

In two previous works [11, 12], the super-unified MSSM was assumed and a number of constraints stemming from direct experimental searches for SUSY, CLEO bounds on $\text{BR}(b \rightarrow s\gamma)$, relic abundances of the lightest SUSY particle, etc. were assumed. What remained of the original parameter space of the super-unified models was called the Constrained Minimal Supersymmetric Standard Model (CMSSM). The natural question is, then, what ranges of values for R_b are predicted for solutions consistent with the CMSSM? In fact, one finds that the constraints of the CMSSM force R_b to lie below approximately 0.2166, some 1.5σ below the reported R_b for $M_t = 174 \, \text{GeV}$. Fig. 1 shows the CMSSM's "line of closest approach" to the experimentally measured value of R_b. This closest approach line does not enter the 1σ area, and therefore CMSSM cannot bring LEP's measurement of R_b into agreement with FNAL's measurement of M_t; conversely, one could say that the CMSSM predicts R_b below the line given in Fig. 1.

Why is the CMSSM incapable of producing larger values for R_b? Naively, one would expect contributions from supersymmetric masses $\sim m_W$ to have a large contribution to the $Z \rightarrow b\bar{b}$ partial width just as the W boson itself has. As described earlier, there exist interactions of the charginos with top squarks and quarks which are large, proportional to

the top Yukawa coupling (h_t), and enhance R_b. Similar contributions exist at large $\tan\beta$ for the neutral Higgs bosons with bottom quarks and squarks, proportional to the $\tan\beta$-enhanced bottom Yukawa coupling. These interactions could lead to R_b consistent with experiment. We now summarize the reasons that they do not.

The coupling of the top quarks to charginos and top squarks, proportional to the large top Yukawa coupling, is given by $\mathcal{L} = h_t \bar{t} \tilde{H}^\pm \tilde{t}_R$. In order to maximize the impact of this coupling, we need a light top squark with a significant right-handed component and a light chargino with a significant higgsino component. These are both difficult requirements for the CMSSM, and they are quite impossible to satisfy simultaneously.

Within CMSSM, the choice of common scalar masses means that $m_{\tilde{t}_R}$ is invariably smaller than $m_{\tilde{t}_L}$ due to the running of the RGE's from the GUT scale down to the weak scale. Yet the resulting $m_{\tilde{t}_1}$ is rarely smaller than m_W unless we impose large mixing between the $\tilde{t}_L$ and $\tilde{t}_R$ eigenstates. This in turn means that $\tilde{t}_1$ will have a significant $\tilde{t}_L$ component which does not couple to the charginos with h_t.

The lightest chargino is even more troublesome. The chargino mass matrix depends on M_2, μ and $\tan\beta$. With radiative breaking μ scales roughly as $\max\{m_{1/2}, m_0\}$ and thus generally dominates the chargino matrix (for a discussion, see Ref. [11]). This means that the charginos with significant higgsino components are heavy, generally well above m_W, and even though they couple with a factor of h_t, these contributions will be kinematically suppressed. Solutions with a chargino as light as $46\,\text{GeV}$ can be obtained, but since they will be almost all $\widetilde{W}^\pm$ they do not couple as h_t.

When we try to simultaneously satisfy the two requirements of a light $\tilde{t}_1 \simeq \tilde{t}_R$ and a light $\chi^\pm \simeq \tilde{H}^\pm$, we find that the CMSSM is incapable of providing a solution. The two requirements actually push solutions in different, incompatible directions. Large mixing in the top squark sector generally requires μ (or m_0) to be large, while light higgsino-like charginos require μ (thus m_0 also) to be small.

In the high $\tan\beta$ region, the additional contributions coming from the pseudoscalar–bottom squark loops are likewise suppressed. The CMSSM generally produces heavy bottom squarks and also heavy pseudoscalars, so that their contributions will be small and cannot bring the CMSSM into agreement with experiment.

For these reasons, and after much numerical work, we can conclude that the CMSSM is incompatible (to 1.5σ) with the experimental measurements of R_b and M_t. Those who wonder how easy it is to vary SUSY parameters in order to fit data should note this result. Clearly, the CMSSM's ability to fit experiment *for this particular observable* is inextricably tied to the Standard Model's since the predictions are essentially the same.

What of models which unify the gauge couplings but not the soft-breaking parameters? One may use a "bottom-up" approach in building models at the GUT scale, starting from the constraints of the low-energy theory. Of particular interest is the case motivated by $t-b-\tau$ Yukawa coupling unification in SO(10). Here one finds that large $\tan\beta$ is necessary, which could be consistent with R_b given the increase in $R_b^{\max}$ that occurs at very large $\tan\beta$ (see Fig. 2). However, the RGE's for the soft masses can be analytically solved in this (pseudo-fixed point) limit [13], and are consistent with a tree-level $m_{A^0}^2 > 0$ only if $\mu^2 \gtrsim 3M_{1/2}^2$. In this case, the light charginos/neutralinos are predominantly wino/bino, not higgsino, and R_b will approach the SM value. Therefore, these SO(10)-type models appear to not be able to yield R_b within 1σ of experiment, regardless of the GUT-scale structure

of the soft-breaking terms.

5   Conclusions

We have explicitly demonstrated a type of No-Lose Theorem for direct observation of SUSY partners at either LEP II or upgraded Tevatron. The central assumption for this theorem is that the actual values of R_b and M_t lie at or above their 1σ lower bounds.

By coupling these measurements to the requirement of perturbative validity of the Yukawa couplings up to a GUT-like scale, we derived bounds on charginos and top squarks leading one to conclude that one or both should be directly observed at LEP II with $\sqrt{s} > 140\,\text{GeV}$, and at an upgraded (in luminosity) Tevatron, for any SUSY breaking and any (perturbative) $\tan\beta$. We also pointed out the existence of interesting bounds on $\tan\beta$ if the lower bound on charginos could be pushed up a little from its current value. Of course, these bounds may never be saturated. Other observables, such as the ρ-parameter, the forward-backward b-asymmetry (A_{FB}^b), or $B^0 - \bar{B}^0$ mixing (χ_B) could significantly tighten our upper bounds; we are currently examining this issue. However, we have checked that full spectra can be found which are consistent with both the experimental value for R_b and a cosmological relic density of $\Omega \simeq 1$.

Finally, we explained the inability of the simplest class of super-unified models to explain the R_b discrepancy. We found that these models are strongly decoupled from the decay of $Z \rightarrow b\bar{b}$ despite their often low mass scales, so that their prediction for R_b bears little difference from that of the SM. We are currently exploring the question of what kinds of GUT models can be found which *are* consistent with R_b. In particular, one wishes to know what hierarchies of soft-breaking masses are required at the GUT scale to replace the assumption of common masses. Surprisingly, we have found that it is extremely difficult to construct a unified theory for any set of soft-breaking parameters if $\tan\beta$ is in the range required by unification of all third generation Yukawa couplings, as in SO(10) models. The solution to this problem may lead to progress in understanding the mechanism of soft SUSY breaking and to the breaking of gauge symmetries at the GUT scale, assuming the experimental R_b discrepancy persists over time.

Acknowledgements

Appendix

Supersymmetric corrections to the $Zb\bar{b}$ vertex are m_t and $m_b \tan\beta$ dependent, and can be quite large. In this appendix we present the calculation of the shift in R_b due to supersymmetric contributions from charged Higgs–top quark loops, chargino–top squark loops, and

$$\nabla_b^{\text{susy}} = 2 \text{ Re } \frac{}{\left| \right|^2}$$
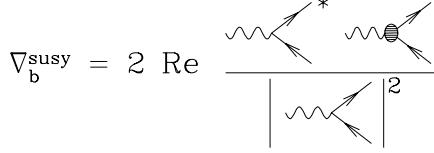
Figure 4: Feynman diagram representation of ∇_b^{susy}.

neutralino–bottom squark loops. Our calculations are in agreement with those of Ref. [6], and in this appendix we follow the notation of this reference as closely as possible. However, we attempt to clear up possible confusion by presenting the equations strictly in terms of ordinary Passarino-Veltman functions with full arguments.

The expansion of R_b can be separated into SM contributions and supersymmetric contributions [6]:

$$
\begin{aligned}
R_b &= \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons}) \\
&= R_b^{\text{sm}}(m_t, m_b) + R_b^{\text{sm}}(0,0)[1 - R_b^{\text{sm}}(0,0)][\nabla_b^{\text{susy}}(m_t, m_b) - \nabla_b^{\text{susy}}(0,0)].
\end{aligned}
$$

$R_b^{\text{sm}}(0,0)$ is the SM prediction of R_b for massless top and bottom quarks which we take to be 0.220. The value for $\nabla_b^{\text{susy}}(m_t, m_b)$ is the sum of one-loop interferences with the tree graph divided by the squared amplitude of the tree graph as shown in Fig. 4.

A convenient parameterization of the supersymmetric vertex corrections $\nabla_b^{\text{susy}}(m_t, m_b)$ is

$$\nabla_b^{\text{susy}}(m_t, m_b) = \frac{\alpha}{2\pi \sin^2 \theta_W} \frac{v_L F_L + v_R F_R}{(v_L)^2 + (v_R)^2},$$

where

$$v_L = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W, \quad v_R = \frac{1}{3}\sin^2\theta_W.$$

Loops involving the charged Higgs with the top quark yield the following contributions to F_L and F_R:

$$
\begin{aligned}
F_{L,R}^a &= S_1(m_t, m_{H^\pm})v_{L,R}\lambda_{L,R}^2 \\
F_{L,R}^b &= [S_2(m_t, m_{H^\pm}, m_t)v_{R,L}^{(t)} + m_t^2 S_3(m_t, m_{H^\pm}, m_t)v_{L,R}^{(t)}]\lambda_{L,R}^2 \\
F_{L,R}^c &= S_4(m_{H^\pm}, m_t, m_{H^\pm})(\frac{1}{2} - \sin^2\theta_W)\lambda_{L,R}^2
\end{aligned}
$$

where

$$
\begin{aligned}
v_L^{(t)} &= \frac{1}{2} - \frac{2}{3}\sin^2\theta_W, \quad v_R^{(t)} = -\frac{2}{3}\sin^2\theta_W \\
\lambda_L &= \frac{m_t \cot\beta}{\sqrt{2}m_W}, \quad \lambda_R = \frac{m_b \tan\beta}{\sqrt{2}m_W}.
\end{aligned}
$$

Loops involving charginos and top squarks yield the following contributions to F_L and F_R:

$$F^a_{L,R} = \sum_{i,j} S_1(m_{\chi^\pm_i}, m_{\tilde{t}_j}) v_{L,R} \Lambda^{*L,R}_{ji} \Lambda^{L,R}_{ji}$$

$$F^b_{L,R} = \sum_{i,j,k} S_4(m_{\tilde{t}_i}, m_{\chi^\pm_k}, m_{\tilde{t}_j})(\frac{2}{3}\sin^2\theta_W \delta_{ij} - \frac{1}{2}T^*_{i1}T_{j1}) \Lambda^{L,R}_{ik} \Lambda^{*L,R}_{jk}$$

$$F^c_{L,R} = \sum_{i,j,k} [S_2(m_{\chi^\pm_i}, m_{\tilde{t}_k}, m_{\chi^\pm_j})O^{R,L}_{ij} + m_{\chi^\pm_i} m_{\chi^\pm_j} S_3(m_{\chi^\pm_i}, m_{\tilde{t}_k}, m_{\chi^\pm_j})O^{L,R}_{ij}] \Lambda^{L,R}_{ki} \Lambda^{*L,R}_{kj}$$

where

$$\Lambda^L_{ij} = T_{i1}V^*_{j1} - \frac{m_t}{\sqrt{2}m_W \sin\beta}T_{i2}V^*_{j2}$$

$$\Lambda^R_{ij} = -\frac{m_b}{\sqrt{2}m_W \cos\beta}T_{i1}U_{j2}$$

$$O^L_{ij} = -\cos^2\theta_W \delta_{ij} + \frac{1}{2}U^*_{i2}U_{j2}$$

$$O^R_{ij} = -\cos^2\theta_W \delta_{ij} + \frac{1}{2}V_{i2}V^*_{j2}.$$

We follow the conventions of reference [15] for the chargino mixing matrices U and V defined in the $\{\widetilde{W}^+, \widetilde{H}^+\}$ basis.

Loops involving neutralinos and bottom squarks yield the following contributions to F_L and F_R:

$$F^a_{L,R} = \sum_{i,j} S_1(m_{\chi^0_i}, m_{\tilde{b}_j}) v_{L,R} \Lambda^{*L,R}_{ji} \Lambda^{L,R}_{ji}$$

$$F^b_{L,R} = \sum_{i,j,k} S_4(m_{\tilde{b}_i}, m_{\chi^0_k}, m_{\tilde{b}_j})(\frac{1}{2}B^*_{i1}B_{j1} - \frac{1}{3}\sin^2\theta_W \delta_{ij}) \Lambda^{L,R}_{ik} \Lambda^{*L,R}_{jk}$$

$$F^c_{L,R} = \sum_{i,j,k} [S_2(m_{\chi^0_i}, m_{\tilde{b}_k}, m_{\chi^0_j})O^{R,L}_{ij} + m_{\chi^0_i} m_{\chi^0_j} S_3(m_{\chi^0_i}, m_{\tilde{b}_k}, m_{\chi^0_j})O^{L,R}_{ij}] \Lambda^{L,R}_{ki} \Lambda^{*L,R}_{kj}$$

where

$$\Lambda^L_{ij} = \frac{1}{\sqrt{2}}(\frac{1}{3}N^*_{j1}\tan\theta_W - N^*_{j2})B_{i1} - \frac{m_b}{\sqrt{2}m_W \cos\beta}N^*_{j3}B_{i2}$$

$$\Lambda^R_{ij} = \frac{\sqrt{2}}{3}\tan\theta_W N_{j1}B_{i2} - \frac{m_b}{\sqrt{2}m_W \cos\beta}N_{j3}B_{i1}$$

$$O^L_{ij} = \frac{1}{2}N^*_{i3}N_{j3} - \frac{1}{2}N^*_{i4}N_{j4}$$

$$O^R_{ij} = -O^{*L}_{ij}.$$

Again, we follow the conventions of reference [15], and define the neutralino mixing matrix N in the $\{\widetilde{B}, \widetilde{W}^3, \widetilde{H}_d, \widetilde{H}_u\}$ basis. Note that this convention is different from that of Ref. [6] which chose to diagonalize the neutralinos in the $\{\widetilde{B}, \widetilde{W}^3, \widetilde{H}_u, \widetilde{H}_d\}$ basis, thereby interchanging N_{i3} and N_{i4} in the above equations.

The S_n functions are defined in terms of the non-divergent parts of Passarino-Veltman scalar integral functions [16]:

$$
\begin{aligned}
S_1(m_1, m_2) &= B_1(-m_b^2; m_1^2, m_2^2) \\
S_2(m_1, m_2, m_3) &= -\tfrac{1}{2} + [2C_{24} - m_Z^2 C_{12} - m_Z^2 C_{23}](-m_b^2, -m_b^2, -m_Z^2; m_1^2, m_2^2, m_3^2) \\
S_3(m_1, m_2, m_3) &= C_0(-m_b^2, -m_b^2, -m_Z^2; m_1^2, m_2^2, m_3^2) \\
S_4(m_1, m_2, m_3) &= -2C_{24}(-m_b^2, -m_b^2, -m_Z^2; m_1^2, m_2^2, m_3^2).
\end{aligned}
$$

All logs encountered in the scalar integrals are made dimensionless by inserting the 't Hooft mass μ_R. Note that our S_2 function contains C_{12}. In Ref. [6], the calculation of R_b is presented in terms of reduced Passarino-Veltman functions [17]. However, the translation there of the $\mathbf{c}_6$ reduced Passarino-Veltman function leaves one to believe that S_2 contains C_{11} rather than the correct C_{12}.

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